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Radiation and lag errors of the Meteorological Office radiosonde and the diurnal variation of upper air temperature

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Summary

Radiation errors of the M.O. radiosonde have been obtained from calculations of the rates of absorption of radiation and of heat loss by forced convection. The effect of the heating of the radiation shield on the temperature of the air reaching the thermometer element becomes very large at high levels. The errors due to direct solar radiation and to radiation reflected and emitted from below an ascending sonde have been calculated for different solar altitudes and for heights up to 30 km.

At solar altitudes between 40° and 60° the errors amount to about 1°, 2.5° and 9°C at 5, 15 and 25 km. respectively. Observed differences in readings from ascending and rapidly descending sondes indicated that the error at 25 km. was within about 7 per cent of the calculated value.

Lag coefficients have been derived from the heat transfer coefficients and the calculated value for ground level is in good agreement with laboratory measurements.

The errors of the radiosonde can account for the whole of the observed differences between day and night observations up to 15 km. Above that height, however, the differences are appreciably greater than the errors and suggest that the daily range of temperature variation over the British Isles in spring and autumn is about 1.5°C at 20 km. and about 3°C at 25 km.

1. Introduction. In M.R.P. 545 the writer described a method of calculating the radiation and lag errors of the Meteorological Office radiosonde, using the theory of heat transfer by forced convection over a flat plate. The numerical results derived in the report were based on assumed values of the absorption factors of a thermometer element and radiation shield with highly polished surfaces. In the present paper measured values, very different from the assumed values but representative of temperature units taken from stock, have been used.

The opportunity has also been taken to use more appropriate heat transfer formulae and to take into account the swinging of the radiosonde, multiple reflections in the temperature unit and the effect of radiation reflected and emitted from the ground or from cloud. Experiments have been made to confirm some of the calculations and to compare some modifications of the radiation shield.

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2. Factors affecting radiation errors. The possible effects of radiation on the temperature unit can be more readily understood by reference to the drawing of the element and shield reproduced in Fig.1A. The temperature element itself is made of polished steel/invar bimetal, 0.25 mm. thick, rolled into the shape of a hollow open-ended cylinder. In the present form of the shield, which is made of thin polished aluminium, the lower half round the element is double-walled but the inner wall is extended upwards by a rectangular duct. This arrangement was adopted after many variations had been tried out by Dymond (1947). Like all radiation shields it is necessarily a compromise, as the following considerations will show.

(a) At high solar elevations there is a possibility of solar radiation falling directly on to the element if the top opening of the shield is such as to allow a free flow of air.

(b) For the same reason radiation may reach the element after multiple reflection inside the shield.

(c) The shield itself may absorb sufficient radiation to warm appreciably the air flowing through it. (The effect of heat re-radiated from the shield to the element can be shown to be negligible).

(d) There is also the possibility of radiation reflected or emitted from the earth's surface or from clouds being strong enough to heat the temperature unit appreciably.

It is clear that the form of shield which is the more effective in preventing (a) and (b) is likely to be less effective against (c), and vice versa.

Although laboratory experiments to measure the effects of radiation under ground level conditions can be devised it is more difficult to extend such experiments to the conditions to which the radiosonde is subjected at high levels. However, the form of the element and shield adopted for the M.O. radiosonde is such as to enable the effects to be calculated by using heat transfer theory.

3. Application of theory of heat transfer. The extent to which the temperature T of a thermometer element with a heat capacity C differs from that of the air reaching it is governed by the heat transfer equation comprising the following terms:-

$$C\,dT/dt = Q + H + K \qquad \text{.....(1)}$$

$C\,dT/dt$ being the net rate of transfer of heat and the other terms representing the rates of transfer, to or from the element, due to radiation, convection and conduction respectively. Since the supports of the radiosonde bimetal element are of very small cross-section the effect of conduction is relatively small and we can neglect K without appreciably affecting the final result. This has been confirmed by the fact that insulating the support of the thermometer element from the frame of the unit made no appreciable difference to its rate of response.

The term Q is the sum of the effects due to direct solar radiation, to radiation reflected from surfaces below the sonde and to long wave radiation emitted from below. The term H is the product of the heat transfer coefficient q and the temperature difference between the absorbing body and the air $(T-T_a)$.

Equation (1) can now be re-written in terms of this temperature difference:

$$T-T_a = \frac{Q}{q} - \frac{C}{q}\,\frac{dT}{dt} \qquad \text{.....(2)}$$

/The

The term Q/q is the temperature rise due to radiation, which is thus obtainable if we know the rate of absorption of radiation and the heat transfer coefficient. The last term in equation (2) is the lag error due to thermal inertia of the body, the ratio C/q being the lag coefficient.

In calculating the radiosonde errors it is necessary to distinguish between (a) the heating of the bimetal element itself and (b) the heating of the radiation shield and its effect on the air flowing through it. It is only the heating of the inner shield, with its extension beyond the outer shield, which is important, and, in what follows, any reference to the shield should be taken to mean the inner shield unless otherwise stated. The two following sections explain how the heat transfer coefficients and the rates of absorption of radiation are derived.

4. Heat transfer by forced convection. The heat transfer coefficient of a body in a moving fluid at a different temperature is given by:-

$$q = Nu\ k\ S/D \qquad \text{.....(3)}$$

where Nu is the Nusselt number, k the thermal conductivity of the fluid, S the area exposed to convective heat transfer and D the characteristic dimension of the body. The Nusselt number is the ratio of the rate of convective heat transfer to the rate of transfer by conduction through the fluid and the values of Nu appropriate to the form and dimensions of the body and the mass rate of fluid flow can be calculated from empirical formulae given in text-books such as that of Fishenden and Saunders (1950). The formulae given in Table I have been used in the present investigation. Re is Reynolds' number given by $v\rho D/\mu$, where v is the rate of air flow, ρ the air density and μ its dynamic viscosity.

Table I

Formulae for Nusselt numbers

Form of surface	Characteristic dimension	Re	Nu
Flat plate	Length in direction of flow.	$< 2 \times 10^4$	$0.59\ Re^{0.5}$
		$> 2 \times 10^4$	$0.032\ Re^{0.8}$
Duct, with effective diameter d and length l	Effective diameter	$< 2.1 \times 10^3$	$1.45\ (Re.d/l)^{0.33}$
		$> 2.1 \times 10^3$	$0.020\ Re^{0.8}$
Cylinder, perpendicular to air flow	Diameter	10^{-1} to 10^3	$0.32 + 0.43\ Re^{0.52}$
		10^3 to 5×10^4	$0.23\ Re^{0.6}$

The numerical constants in these formulae are dependent on Prandtl's number (given by $C_p\mu/k$, where C_p is the specific heat of the fluid at constant pressure) which has the value 0.71 for air over the range of temperature and pressure in atmospheric conditions up to 30 km.

The flat plate formulae can be applied to obtain the heat transfer coefficient q_e for the inner and outer surfaces of the bimetal element in the shield, the characteristic dimension D being the length in the direction of the air stream through the shield. For the coefficient q_i for the inner surface of the shield the duct formulae are applicable, with the effective diameter as D. The flat plate formulae would be appropriate for the outer surface of the shield if the sonde ascended vertically. Generally, however, it swings at the end of its 40m.

/suspension

suspension with an amplitude of about ± 30°, so there is an air flow perpendicular to the axis of the shield varying between 0 and 10 m/s. The average resultant air flow is about 9.5 m/s at an angle of about 45° to the axis. There is little difference between the heat transfer coefficient q_p, given by the flat plate formula, for an air flow parallel to the axis and the coefficient q_c, given by the cylinder formula, for the same rate of air flow perpendicular to the axis. The mean of q_p and q_c for a rate of flow of 9.5 m/s has therefore been taken as representing the coefficient q_o for the outer surface of the shield.

Table II gives the dimensions used in obtaining the heat transfer coefficients.

Table II

Dimensions used in calculating heat transfer coefficients

Surface	Aerodynamic shape	Effective length	Width (perimeter)	Effective diam.	Surface area
		cm.	cm.	cm.	$cm.^2$
Bimetal element, inner and outer surfaces.	Flat plate	1.6	3.0	-	9.6
Shield, inner surface.	Duct	15	-	3.3*	157
Shield, outer surface, flow parallel to axis.	Flat plate	10	15	-	150
Shield, outer surface, flow perpendicular to axis.	Cylinder	8.5	-	4.7+	127

* Effective diameter of duct of rectangular cross-section
= 4 x cross-section/perimeter

\+ Effective diameter of rod of rectangular cross-section = perimeter/π

The values of the coefficients for a sonde ascending at 6 m/s and swinging through an angle of ± 30° have been calculated for various heights and are given in Table III. With the rate of ascent constant the basic variables are the density, viscosity and thermal conductivity of the air; the values of these used are included in Table III.

Table III

Heat transfer coefficients for sonde ascending at 6 m/s and swinging ± 30°

Unit: cal. sec. $^{-1}$ $^{\circ}C^{-1}$ x 10^{-3}

Height km.	Pressure mb.	Density $10^6\rho$	Dyn. Visc. $10^6\mu$	Thermal Cond. 10^6k	Shield					Air in shield		Element
					q_p	q_c	q_o	q_i	q_s	m C_p	B	q_e
30	12	18	147	49	15.2	15.8	15.5	13.0	25.8	23.8	.420	1.90
25	25	39	146	49	21.9	22.3*	22.1	16.3	36.2	50	.280	2.77
20	55	88	144	48	31.4	34.6*	33.0	20.5	51.7	106	.175	3.95
15	115	184	145	48	46.9	50.8*	48.9	24.1*	71.8	238	.097	5.9
10	264	413	146	49	71	90.7*	81	45.6*	125	520	.084	8.9
5	531	723	163	54	136*	135*	135	75.5*	208	946	.077	12.7
0	1013	1225	179	60	201*	184*	192	112*	301	1530	.071	16.9

/Explanatory

Explanatory notes for Table III:

* The Reynolds' numbers in these cases are above the critical values given in Table I; in the other cases they are below these values.

q_p refers to flow along outer surface parallel to axis.

q_c refers to flow over outer surface perpendicular to axis.

q_o is the mean of q_p and q_c.

q_i refers to flow over inner surface.

q_s = $q_o + mC_p B$ = total transfer coefficient for shield.

mC_p is the mass rate of heat flow in the air through the shield.

B = $(T_a^1 - T_a)/(T_s - T_a)$ i.e. ratio of heating of air in shield to the heating of the shield.

q_e refers to flow over inner and outer surfaces of element.

As there is relatively little variation of viscosity and thermal conductivity with height, especially above 10 km., it is the air density which is almost wholly responsible for any change in the heat transfer, and therefore the radiation error, with height.

In applying the calculated heat transfer coefficients in equation (2) there is a complication arising from the fact that whereas the formulae for the Nusselt numbers for the flat plate and cylinder are based on temperature differences $(T-T_a)$ between the body and the undisturbed air upstream, the formulae for the duct refer to the mean of the differences at the inlet and outlet. Further, since the air temperature in the duct varies across the cross-section, a mean value T_a^1 corresponding to perfect mixing is assumed. It is this temperature which is taken to be that of the air reaching the bimetal element. Applying, then, equation (2) to the case of the shield we have:

$$C_s \frac{dT_s}{dt} = Q_s - q_o (T_s - T_a) - q_i \left[T_s - (T_a + T_a^1)/2\right] \qquad \text{.....(4)}$$

The last term, representing the rate of heat loss by the inner surface, must be equal to the rate of gain of heat by the air passing through the shield, which is $mC_p (T_a^1 - T_a)$, where m is the mass rate of air flow, i.e. $\pi D^2 \rho v/4$, D being the effective diameter of the duct. It follows that:

$$\frac{T_a^1 - T_a}{T_s - T_a} = \frac{2q_i}{2mC_p + q_i} = B \text{ (say)} \qquad \text{.....(5)}$$

thus giving the ratio of the heating of the air to the heating of the shield. It also follows that:-

$$q_i \left[T_s - (T_a + T_a^1)/2\right] = mC_p B (T_s - T_a) \qquad \text{.....(6)}$$

We can therefore now express the heat balance of the shield by:

$$C_s \frac{dT_s}{dt} = Q_s - q_s (T_s - T_a) \qquad \text{.....(7)}$$

where q_s, the total heat transfer coefficient, is given by:

$$q_s = q_o + mC_p B$$

/The

The calculated values of q_o, mC_p, B and q_s are included in Table III. An experimental determination of the value of B was attempted by measuring the temperature of air passing through a heated shield in a wind tunnel. The shield was maintained about 5°C warmer than the air stream, the speed of which was 6 m/s. Measurements were made with the axis of the shield not only in the direction of the air flow but also displaced 30° in horizontal and vertical planes. Temperatures were measured by thermocouples but as the experiments could only readily be made at ground level pressure the temperature differences between the air entering and leaving the shield were very small and there was a large scatter in the individual readings. The mean, however, of 24 measurements gave the value of B as 0.08 $\pm$ 0.08 P.E. which is not inconsistent with the calculated value of 0.071.

5. Rate of absorption of radiation. The radiation term Q in the heat balance equation depends, of course, on the intensity of the incident radiation I, on the absorption factor A of the body and on the orthogonal projection of the area exposed to the beam. The computation of Q is complicated by (a) the swinging of the sonde, (b) shading of the shield or element by other parts of the sonde, and (c) multiple reflections of radiation entering the open ends of the shield.

For the absorption of solar radiation of intensity I by the element or the shield we can write:

$$Q_h = AI \cos h.\ f(S_d, h, A_s) \qquad (8)$$

where h is the solar altitude, S_d the effective vertical area exposed, including that receiving radiation reflected internally, and A_s is the absorption factor of the shield.

The numerical values of f (S_d, h, A_s) are best obtained by geometrical measurement.

The rate of absorption of radiation reflected from the ground or from cloud is given by:

$$Q_m = AI_r \int_{\alpha_1}^{\alpha_2}\int_{\phi_1}^{\phi_2} \cos^2\phi.d\phi.S_m d\alpha \qquad (9)$$

where S_m is the effective vertical area of the shield or element exposed, which may vary with the azimuth and elevation angles, α and ϕ, of the effective portions of the radiating surfaces as "seen" by the sonde. I_r is the intensity of the reflected radiation and is given by:

$$I_r = RI \sin h \qquad (10)$$

where R is the albedo of the reflecting surface and I is the intensity of solar radiation at the surface.

The rate of absorption of radiation Q_l emitted from below is also given by (9) except that the intensity of the long wave radiation I_l and the appropriate absorption factor A_l must be used. The intensity I_l is given by

$$I_l = \sigma (T_o^4 - T^4) / \pi \qquad (11)$$

/where

where σ is the Stefan-Boltzmann coefficient (1.37×10^{-12} cal.cm.$^{-2}$sec.$^{-1}$°C^{-4}) and T_o and T are the temperatures of the emitting surface (cloud or ground) and the sonde. The element and shield themselves emit a negligible amount of radiation to their immediate surroundings.

The angular limits α_1, α_2, ϕ_1, ϕ_2, in (9) are governed by the extent of the shading of the element and the shield by other parts of the sonde. For evaluating Q_m and Q_l the integrations with respect to $d\phi$ have been done in 10° steps, and the effect of swinging of the sonde has been taken into account. The intensity of solar radiation is taken to be 0.032 cal. cm.$^{-2}$ sec.$^{-1}$ at the top of the atmosphere but in the final results it is adjusted for air mass appropriate to the solar altitude and the height of the sonde by means of a graph provided by Vaisala (1948). In evaluating I_r a value of 0.35 for R has been taken as representing average conditions of the earth's surface and cloud cover for the British Isles. The long wave radiation I_l has been calculated for a surface temperature T_o of 288°K and a sonde temperature T of 220°K; these conditions cover most soundings in the British Isles except at heights below 10 km.

The absorption factors of the temperature element and the shield were measured by the National Physical Laboratory at various wave lengths between 0.3 and 10 microns for both normal and 45° incidence, a random sample of each material being taken from stock. From these measurements and from the spectral distribution of solar energy the absorption factors were found to be 0.33 for the aluminium shield and 0.75 and 0.60 for the two sides of the bimetal element. A mean of 0.67 has been used for the element. The values for 45° incidence did not differ from those at normal incidence by more than 0.02. The N.P.L. values for a wave length of 10 microns, viz. 0.03 for the shield and the mean of 0.18 for the element have been used for the absorption factors for the long waves. It must be borne in mind that there may be appreciable differences between individual samples, depending on the state of their surfaces. For a white card shield used in some experimental soundings to be described later, factors, based on published data, of 0.25 for solar radiation and 0.95 for long waves have been used.

Calculated rates of absorption of radiation by the shield and by the element inside it for different solar altitudes are given in Table IV.

Table IV

Rate of absorption of radiation in cal.sec.$^{-1}$ x 10^{-3}

Solar altitude	h =	10°	30°	50°	70°	90°
Shield						
Direct solar:	Q_h	543	597	543	422	415
Reflected from below:	Q_m	32	93	141	174	185
Emitted from below:	Q_l	8	8	8	8	8
Total:	Q_s	583	698	692	604	608
Element						
Direct solar:	Q_h	0.9	4.1	7.8	9.4	12.2
Reflected from below:	Q_m	0.7	2.0	3.0	3.7	4.0
Emitted from below:	Q_l	0.5	0.5	0.5	0.5	0.5
Total:	Q_e	2.1	6.6	11.3	13.6	16.7

Note: The figures for the absorption of reflected radiation are for an albedo 0.35 and those for long wave radiation refer to a surface at 288°K with the sonde at 220°K.

/The

The values for direct radiation take account of the swinging of the sonde through an angle of 30°. This swinging makes no appreciable difference to the amount of radiation reaching the sonde from below. The values are plotted in Fig.2 for comparison with values relating to a sonde remaining vertical. The curves are similar in shape to those given by Vaisala (1941) for his design of sonde.

The effect of radiation emitted from below is quite negligible in the case of the shield, but the contribution by the reflected radiation to the total absorption increases from near zero at low solar altitudes to 30 per cent at 90°. In the case of the element the very small amount of radiation absorbed at low solar altitudes is roughly half due to the direct radiation entering the upper opening of the shield and half to radiation reflected or emitted from below. But at high solar altitudes the direct radiation contributes about three-quarters and reflected radiation one quarter to the total, the effect of the emitted radiation then being less than five per cent. It must be remembered that at low solar altitudes the intensity of the sun's radiation is reduced appreciably below the value 0.032 cal. $cm.^{-2}sec.^{-1}$ assumed in Table IV. At solar altitudes above 20° and heights about 10 km. the reduction is less than 10 per cent.

6. Calculated radiation errors. The values of q_s, q_e, B, Q_s and Q_e given in Tables III and IV enable us to obtain the radiation terms Q_s/q_s and Q_e/q_e in the equations:

$$T_s - T_a = \frac{Q_s}{q_s} - \frac{C_s}{q_s}\frac{dT_s}{dt} \quad \text{and} \quad T_e - T_a^1 = \frac{Q_e}{q_e} - \frac{C_e}{q_e}\frac{dT_e}{dt} \qquad \text{.....(12)}$$

But whereas Q_s/q_s gives the radiational heating of the shield above the free air temperature, Q_e/q_e only gives the heating of the element above the temperature T_a^1 of the air in the shield. The definition of B in equation (5), however, enables us to eliminate T_a^1 and obtain the heating of the element above the free air temperature, thus:

$$T_e - T_a = \frac{Q_e}{q_e} + \frac{BQ_s}{q_s} - \frac{C_e}{q_e}.\frac{dT_e}{dt} - \frac{BC_s}{q_s}.\frac{dT_s}{dt} \qquad \text{.....(13)}$$

The terms $(Q_e/q_e + BQ_s/q_s)$ together represent the total radiation error of the bimetal element in the shield. The calculated values of this error for a selection of heights and solar altitudes are given in Table V, which also includes values for the heating of the shield and the air in it.

/Table V

Table V

Calculated radiation errors of the M.O. radiosonde

(Combining the effects of direct solar radiation, reflected radiation from earth and/or cloud with average albedo 0.35, and long wave radiation emitted from the earth and/or cloud).

	Solar Altitudes	Height km.	10°	30°	50°	70°	90°
					°C		
(a)	Heating of shield Q_s/q_s	30	22.6	27.1	26.9	23.4	23.6
		20	11.3	13.5	13.4	11.7	11.8
		10	4.7	5.6	5.6	4.8	4.9
		0	1.9	2.3	2.3	2.0	2.0
(b)	Heating of air in shield BQ_s/q_s	30	9.6	11.4	11.3	9.9	10.0
		20	2.0	2.4	2.3	2.0	2.1
		10	0.4	0.5	0.5	0.4	0.4
		0	0.1	0.2	0.2	0.1	0.1
(c)	Error of element with respect to air in shield Q_e/q_e	30	1.1	3.5	6.0	7.2	8.8
		20	0.5	1.7	2.9	3.4	4.2
		10	0.2	0.7	1.3	1.5	1.9
		0	0.1	0.4	0.7	0.8	1.0
(d)	Error of element with respect to free air $Q_e/q_e + BQ_s/q_s$	30	10.6	14.9	17.3	17.1	18.8
		25	5.2	7.7	9.3	9.5	10.7
		20	2.4	4.0	5.1	5.3	6.2
		15	1.1	1.9	2.7	3.0	3.5
		10	0.5	1.1	1.7	1.8	2.2
		5	0.3	0.7	1.1	1.2	1.4
		0	0.1	0.4	0.7	0.7	0.9

NOTE: In (a), (b) and (c) the intensity of solar radiation is taken to be 0.032 cal.cm.$^{-2}$sec.$^{-1}$. In (d) account is taken of the effect of air mass on the intensity.

It will be seen that for solar altitudes about 50° the heating of the air in the shield is responsible for about 20 per cent of the radiation error near the ground, 30 per cent at 10 km. and 65 per cent at 30 km. The contributions are higher still at lower solar altitudes since the angle of incidence of solar radiation on the shield is nearer to normal incidence. The remainder of the error is, of course, due to radiation falling on the element itself, either directly or after reflection by the inner surface of the shield.

7. Lag errors. The lag in temperature of the shield and the element is represented by the last terms in equations (12), C/q being the lag coefficient which is usually written as λ. Since C is readily obtained from the mass of the body and its specific heat we can obtain λ from the calculated value of q. The lag coefficient can also be measured in the laboratory (where radiational exchange can be made negligible) by observing the time required for an initial temperature difference between the thermometer element (or shield) and the air to be reduced in the ratio 1/e. In such conditions

$$T - T_a = (T_o - T_a)\exp - \frac{t}{\lambda} \qquad \text{(14)}$$

where $T = T_o$ at $t = 0$. This measurement of λ therefore affords a check on the calculated heat transfer coefficient.

/For

For the bimetal element without its shield the measured value of λ_e in an air flow of 6 m/s at ground level pressure was 7.3s. Under the same conditions the calculated value of q_e of 16.9×10^{-3} (from Table III), with C = 0.116, gives λ_e = 6.9s. The agreement is reasonable.

When the thermometer element is in its shield, however, its lag behind the free air temperature will be affected by the lag of the shield itself and by the extent to which the shield heats the air reaching the element. The total lag error is, in fact, given by the last two terms of equation (13), but it is not readily evaluated since dT_s/dt is not normally measured. The lag of the element in the shield can, however, be measured in the laboratory. From equation (14) and equations (12) for the shield, together with the definition of B, it can be shown that for a sudden temperature difference in the absence of radiational exchange:

$$\frac{dT_e}{dt} = \frac{1}{\lambda_e}\left[T_e - T_a - B(T_o - T_a)\exp\frac{-t}{\lambda_s}\right] \qquad (15)$$

and that:

$$T_e - T_a = (T_o - T_a)\left(\exp\frac{-t}{\lambda_e}\right)\left[1 + \frac{B\lambda_s}{\lambda_s - \lambda_e}\left\{\exp\left(\frac{t}{\lambda_e} - \frac{t}{\lambda_s}\right) - 1\right\}\right] \qquad (16)$$

Now when λ_s is not very different from λ_e, which is the case with the radiosonde unit, and when the measurements are made with t not greatly exceeding λ_e, this simplifies to:

$$T_e - T_a = (T_o - T_a)\exp-\frac{t(1 - B)}{\lambda_e} \qquad (17)$$

In other words the lag coefficient of the shielded element λ_{es} is 1/(1-B) times that of the unshielded element and the total lag error can now be written:

$$\lambda_{es}\frac{dT_e}{dt} \qquad \text{or} \qquad \frac{\lambda_e}{1-B}\cdot\frac{dT_e}{dt}$$

The laboratory measurement of λ_{es} under the same conditions as for λ_e gave a value of 7.6s. The calculated value given by $C_e/q_e(1-B)$ is 7.4s.

It is difficult to measure the lag coefficient in laboratory conditions corresponding with the normal rate of ascent at very low air densities, but it is reasonable to expect that the method of calculation which has been found to give satisfactory agreement at ground level should also be satisfactory for high level conditions. Table VI gives the lag coefficients calculated for various heights and the normal rate of ascent of 6 m/s. The values of the heat capacities used were 0.116 at 288°K and 0.106 at 220°K for the element and 2.42 and 2.16, at these temperatures, for the shield.

/Table VI

Table VI

Lag coefficients and errors (rate of ascent 6 m/s)

Height	Coefficients, in seconds λ_e	λ_s	λ_{es}	Typical errors (°C) night	day
30	56	84	97	-0.6	-1.6
25	38	60	54	-0.3	-0.7
20	27	42	33	-0.2	-0.3
15	18	30	20		
10	11.9	17.3	12.9	+0.5	+0.5
5	8.7	11.0	9.5	+0.3	+0.3
0	6.9	8.0	7.4	+0.3	+0.3

NOTE: The lag errors are for a temperature gradient of -6°C/km. below 15 km. and +1°C/km. above 15 km. The daytime errors take account of the rate of change of temperature of the element by radiational heating for solar altitude 40°.

The table also includes the lag errors for typical rates of change of indicated temperature in night and day soundings. In the troposphere the errors are the same sign as, and about one third the magnitude of, the radiation errors. In the stratosphere they are very small in comparison with the radiation errors and are of opposite sign.

8. Observed errors at high levels. Some comparisons of temperatures indicated at the same height by a radiosonde ascending at the normal rate and descending at a much higher rate have afforded some confirmation of the existence of the large radiation errors at high levels. Such comparisons were made in five soundings at Downham Market (52°36'N., 0°24'E.) in September, 1950 when the balloons burst at heights near 30 km. and the parachutes failed to open immediately. The sondes were tracked by radar. Fig.3 is a graph of the readings of temperature and height obtained during one of the soundings and shows the sudden fall in the temperature readings during the rapid descent.

In the descent the heating, by the shield, of the air reaching the element is negligible since the latter is near the bottom opening of the shield. The temperature T_d indicated by the element during the descent is therefore given by:

$$T_d - T_a = \frac{Q_e}{q_d} - \frac{C_e}{q_d}\cdot\frac{dT_d}{dt} \qquad \text{(18)}$$

where q_d is now the much larger heat transfer coefficient of the element for the much higher rate of air flow V.

Combining equations (13) and (18) then gives the difference between the ascent and descent temperatures:

$$T_e - T_d = \frac{Q_e}{q_e} + \frac{BQ_s}{q_s} - \lambda_{es}\frac{dT_e}{dt} - \frac{Q_e}{q_d} + \lambda_d\frac{dT_d}{dt} \qquad \text{(19)}$$

This can be evaluated from the calculated values of the radiation errors and the lag coefficients and the observed rates of change of indicated temperatures. The results of the calculations and comparisons are given in Table VII.

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Table VII

Observed and calculated errors (solar altitude 40°)

Date 1950 Sept.	Height km.	Rate of descent m/s	Calculated errors in °C						Ascent - descent (T_e-T_d)		
			Ascent			Descent					
			Radiation	Lag	(T_e-T_a)	Radiation	Lag	(T_d-T_a)	Calc.	Obs.	O-C
12	24.0	41	7.9	0.2	7.7	1.2	-0.7	1.9	5.8	6.1	+0.3
"	21.8	34	6.1	0.3	5.8	1.1	-0.2	1.3	4.5	5.2	+0.7
13	28.2	57	13.4	2.8	10.6	1.4	-1.3	2.7	7.9	8.8	+0.9
"	25.8	47	9.8	1.3	8.5	1.3	-0.6	1.9	6.6	6.0	-0.6
19	28.8	61	14.5	2.0	12.5	1.4	-2.9	4.3	8.2	10.3	+2.1
"	25.2	44	9.1	1.3	7.8	1.3	-1.1	2.4	5.4	5.7	+0.3
20	27.8	55	12.7	1.3	11.4	1.4	-2.1	3.5	7.9	9.4	+1.5
"	24.3	42	8.2	0.5	7.7	1.2	-0.7	1.9	5.8	4.5	-1.3
21	27.1	52	11.6	1.1	10.5	1.3	-1.4	2.7	7.8	8.9	+1.1
"	23.3	38	7.2	0.4	6.8	1.2	-0.2	1.4	5.4	6.0	+0.6
Mean	25.6	47	10.0	1.1	8.9	1.3	-1.1	2.4	6.5	7.1	+0.6

It should be remembered that the observations for each height are single pairs of readings and considerable scatter in the results must be expected. The calculation of the lag error in the descent is probably the most doubtful factor in the computations, since the rapid rate of change of temperature is difficult to measure accurately. However, the mean difference of 0.6°C between the observed and calculated temperature differences can be considered reasonably good and it might well be accounted for by the albedo being different from the value 0.35 assumed in the calculations. The soundings were made with the sky about half clouded but the surface exposed to the sondes through the breaks in the cloud was part grassland and part sea.

9. Comparisons with modified shields. In order to see whether radiation errors could be appreciably reduced by modifying the design of the shield a series of comparisons of two sondes, one standard and one modified, on the same balloons was carried out at Downham Market in September, 1950. The modifications used are shown in Fig. 1. The series also included sondes without any shield, i.e. with the element fully exposed except for partial shading by the sonde. Five or six soundings to high levels were made with each modification. Calculated radiation errors for the standard and for the modified sondes at solar altitudes 40°, are plotted in Fig. 4. The results of the comparisons are given in Fig. 5 in which the points on the graphs represent the means of about 5 observations of the differences (except at the highest levels where fewer were obtained). Probable errors of the mean differences are also indicated. The calculated differences are derived from Fig. 4 and do not include the lag errors, which, however, would not affect the observed differences by more than 0.3°C. The agreement between the observations and the calculations is generally within 1°C. The differences that the modifications make, especially below 20 km., are relatively small. Dymond (1947) found much the same result in some earlier experiments with modified shields. It emphasises the fact, already mentioned, that the design has to be a compromise between conflicting requirements. Thus, reducing the length of the shield, as in modification B, decreases the heating effect of the shield but exposes the element more to direct radiation. Further decrease of the radiation falling on the shield, by extending the outer shield as in modification C, appears to be nullified by decrease in the heat convected from the outer surface of the inner shield. Moreover, neither B nor C can make any difference to the amount of radiation absorbed

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inside the shield. The effect of the heating of the standard pattern shield increases so rapidly at high levels that at 30 km. the radiation error is as large as when the element is fully exposed. The only modification that showed any appreciable improvement - and this only at levels above 20 km. - was D, in which the inner shield was made of white card instead of aluminium. This is probably due to the card having a lower absorption factor for solar radiation and to its being a diffuse reflector and so reflecting less radiation on to the element.

10. The diurnal variation of upper air temperature. If the radiation and lag errors are correctly evaluated the magnitude of the diurnal range of air temperature variation, and its change with height, should be obtainable from the differences between day and night radiosonde observations. In Fig. 6 the curve gives the combined radiation and lag errors for a solar altitude of 40°. In the same graph are plotted four series of mean differences between night and day observations, the latter being made at about this solar altitude. With the exception of those derived from three pairs of high altitude soundings at Downham Market on 15th, 25th and 29th September, 1950, the means are for routine soundings for the six months February to April and August to October combined. Those published by Kay (1951) represent about 1800 observations in 1947 and 1948 at five stations in the British Isles. Piegza's (1947) means are derived from about 700 observations in 1943-6 at Fazakerley (53°28'N., 2°55'W.). The means for the routine soundings at Downham Market represent about 1,400 observations in the period 1943-51. It is clear that up to about 15 km. radiation errors could account for the whole of the day and night differences. Above that height the comparisons indicate that the differences are too big to be accounted for wholly by the errors, if these are correctly assessed, and that there is a diurnal range of about 1.5°C at 20 km. (55 mb.) increasing to about 3°C at 25 km. (25 mb.). We cannot, of course, regard the day and night differences above 22 km. as representative of average conditions since they were based on only three comparisons and although each was made on the same day, at 10h. and 22h., they may well be affected by day to day temperature changes. On the other hand much the same differences above 22 km. were obtained in the means of 15 day and 15 night soundings in September and October which were not made on the same days.

The diurnal ranges at 20 km. and 25 km. indicated by these results are ten times as large as those recently calculated by F.S. Johnson (1953) from the heating due to absorption of solar energy by atmospheric ozone. He used direct measurements of ozone concentration and ultra-violet spectral intensity of sunlight during a rocket flight over New Mexico in June, 1949 and, assuming a constant rate of energy loss, found the diurnal temperature change to have a maximum value of 5.3°C at 48 km. falling to 1°C at 31 km. and 70 km.

Goldie (1951) has suggested the possibility of the diurnal temperature change being due to dynamical causes, in which case the diurnal range should vary approximately inversely with the pressure (or density) and should, therefore, increase exponentially with height. An adiabatic heating of 1.5°C at 20 km. would require a pressure increase of 1.3 mb. above that level and this same increase would also account for the heating of 3°C at 25 km. An inflow of 1 or 2 mb. of air above these levels, with a corresponding outflow at low levels, is not unreasonable. Clearly, however, further information at high levels is required before any safe conclusions about diurnal variation can be drawn.

11. Conclusion. Little advantage is to be gained by introducing any further modifications to the radiation shield as they cannot reduce the radiation errors to negligible proportions. A better course is to use a temperature element for which the ratio of the rates of absorption of radiation and heat convection is so small that the shield becomes unnecessary. The factors favouring this are (a) low absorption factor, (b) small effective surface area exposed to radiation, and (c) large characteristic dimension for the Nusselt number.

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In general, (b) and (c) are mutually antagonistic, but a thin plate which would favour (c) would also favour (b) if its edge could be kept facing the source of radiation. This, of course, is impracticable in a radiosonde and probably the best compromise in a thin wire element perpendicular to the air flow. Calculations show that the maximum radiation error of such an element of diameter 10^{-3}cm. at 30 km. would be only 0.35°C even if its absorption factor were unity. An element of this type, using wire of 1.35×10^{-3}cm. diam., is to be used in the form of a coiled coil in the M.O. radar sonde and it is hoped that the radiation errors will be small enough to allow the magnitude of the diurnal variation of temperature to be determined accurately.

In the meantime, Table V (d) and the lag coefficient λ_{es} given in Table VI can be used for applying corrections to daytime observations obtained with the M.O. radiosonde.

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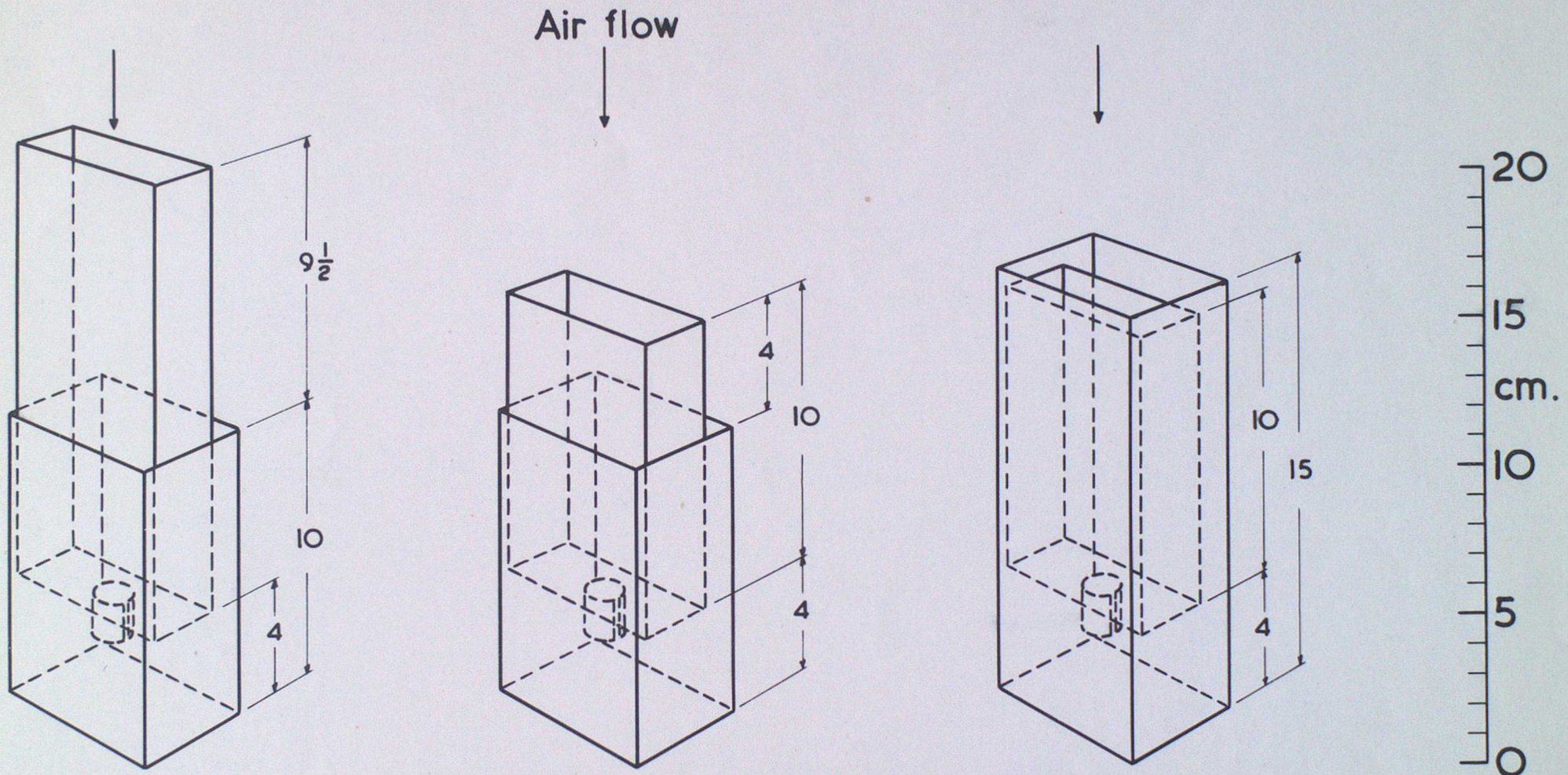
M.R.P. 836



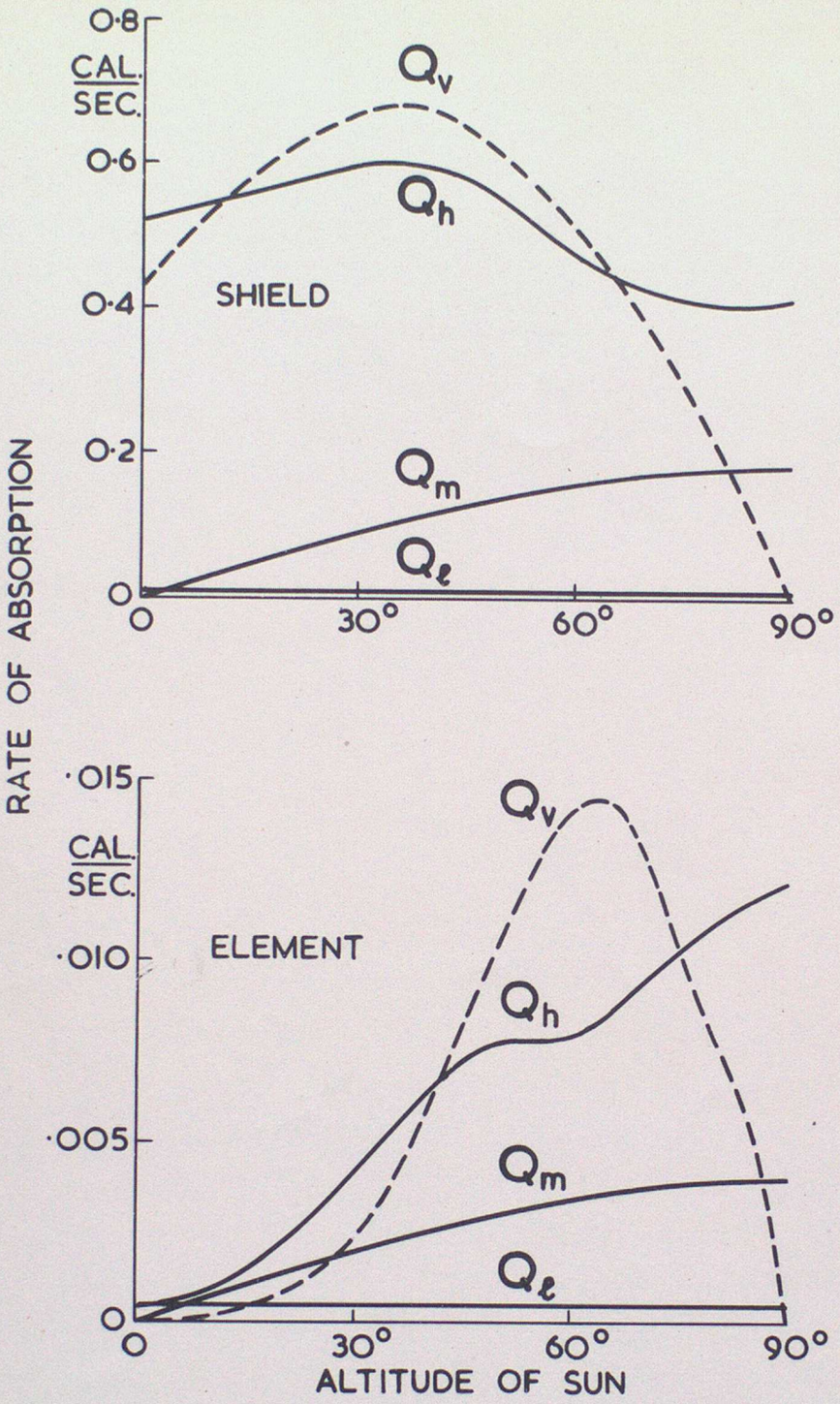
A. Current type

B. Former type

C. Inner shield aluminium

D. Inner shield white card

Fig. 1 Radio sonde radiation shields



Q_v direct solar radiation; sonde vertical

Q_h direct solar radiation; sonde swinging $\pm 30°$

Q_m reflected solar radiation; albedo 0·35

Q_ℓ long wave radiation; earth at 288°K, sonde at 220°K

(Effect of swinging on Q_m and Q_ℓ is negligible)

Fig. 2 Radiation absorbed by element and shield

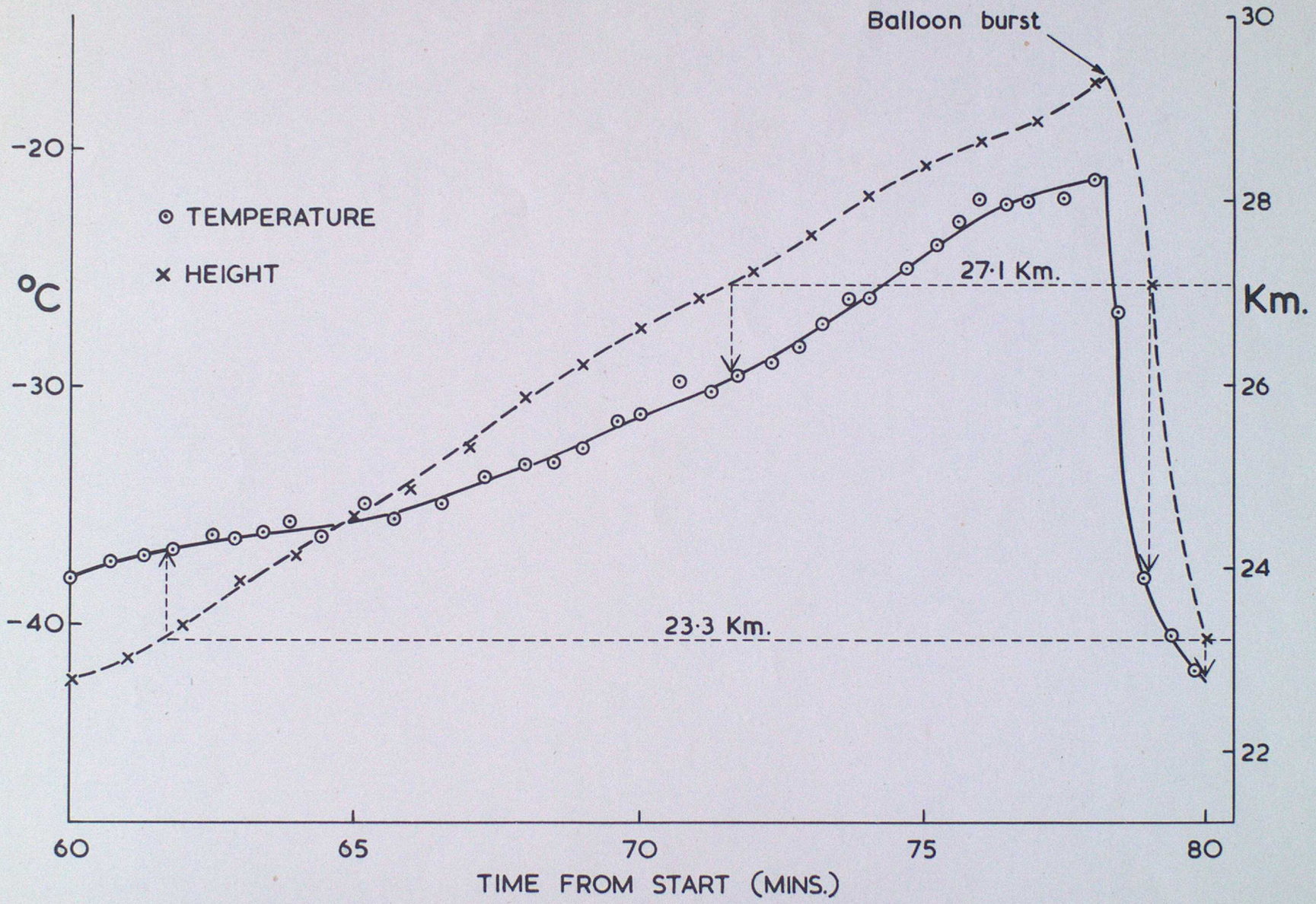


Fig. 3 Temperatures and heights observed in ascent and descent at Downham Market 10h. 21 Sept. 1950

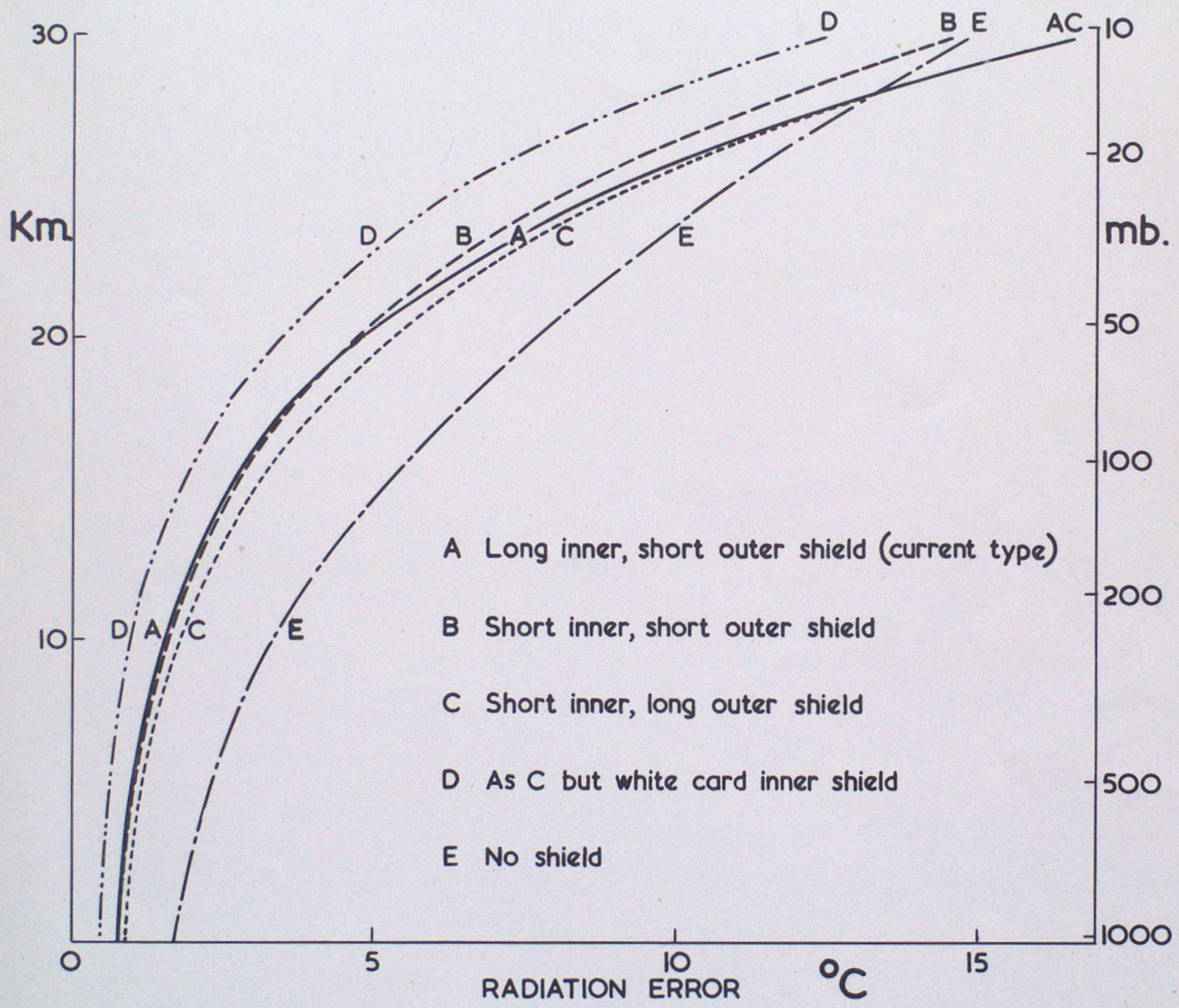


Fig. 4 Calculated radiation errors with standard and modified shields (solar altitude 40°)

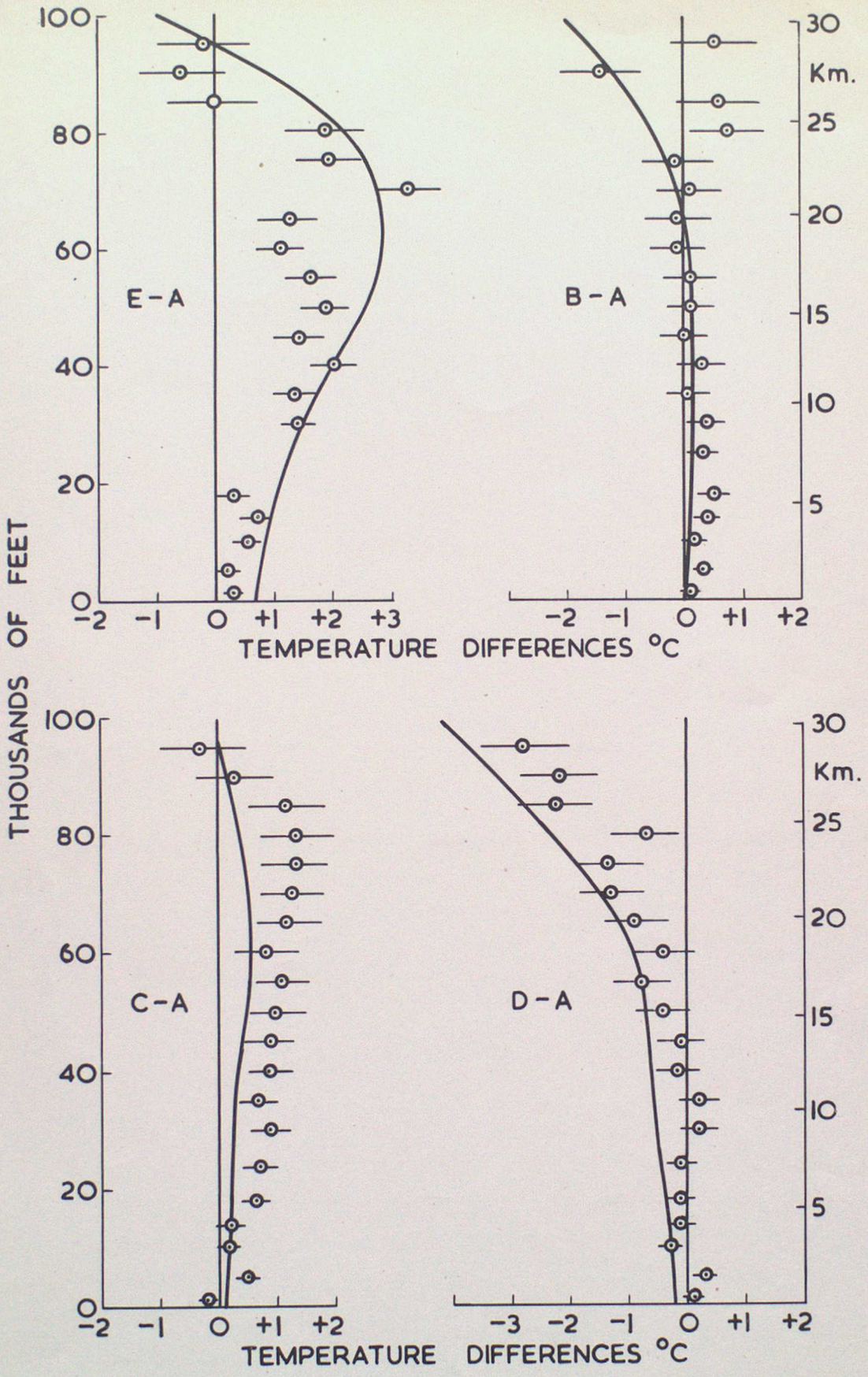


Fig. 5. Observed and calculated temperature differences with standard and modified shields (solar altitude 40°)

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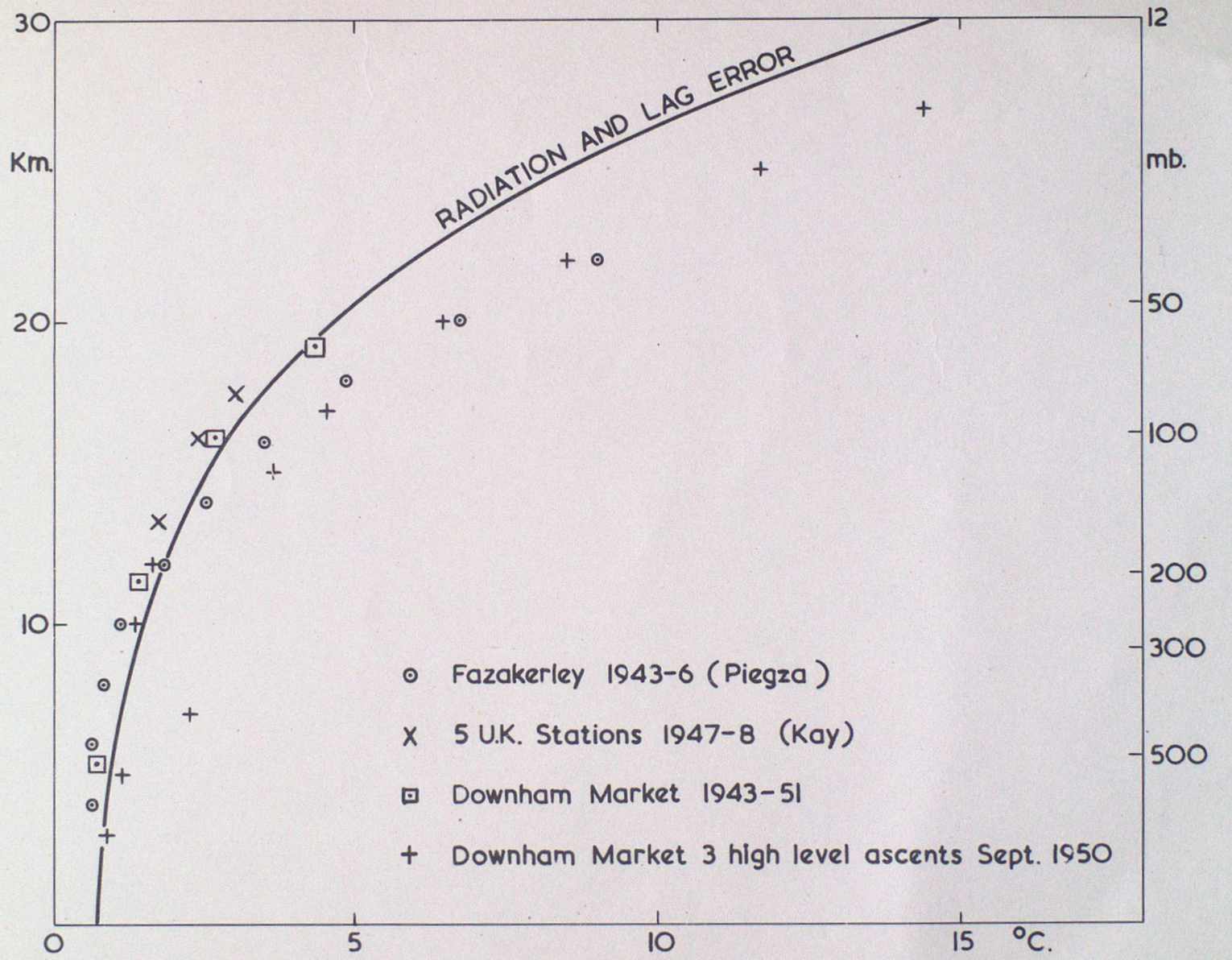

Fig. 6 Day and night temperature differences compared with instrument errors (solar altitude about 40°)